I2200: Digital Image processing

Lecture 6: Image Restoration

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Thanks to G&W website and Lexing Xie for slide materials
Announcement:

- Michael will present HW2 Today.
- HW3 is out today, due on 11/06.
- Peter will present HW3 on 11/07.
- **Midterm Exam: Oct. 24, 2018.**
  - Open notes
  - No electronic device is allowed except calculator
- **Final project:** Send me the following info before **10/31, 2018**
  - The project title and a short description if you choose your own project (**must be image processing related**).
  - Your partner if you want to team with someone.
We have covered …

Image sensing

Image Restoration

Image Transform and Filtering

Spatial Domain processing
Outline

- What is image restoration
  - Scope, history and applications
  - A model for (linear) image degradation
- Restoration from noise
  - Different types of noise
  - Examples of restoration operations
- Restoration from linear degradation
  - Inverse and pseudo-inverse filtering
  - Wiener filters
  - Blind de-convolution
- Geometric distortion and its corrections
Degraded images

- What caused the image to blur?
- Can we improve the image, or “undo” the effects?
Image Restoration

- Image restoration is to "compensate for" or "undo" defects which degrade an image.

- Degradations:
  - motion blur
  - noise
  - camera defocus
  - distortion
  - ...

http://www.owlnet.rice.edu/~elec539/Projects99/BACH/proj2/intro.html
Image Restoration VS Enhancement

- Image enhancement: “improve” an image subjectively.
- Image restoration: remove distortion from image in order to go back to the “original” → objective process.
Image restoration

- started from the 1950s
- application domains
  - Scientific explorations
  - Legal investigations
  - Film making and archival
  - Image and video (de-)coding
  - ...
  - Consumer photography

- related problem: image reconstruction in radio astronomy, radar imaging and tomography

[Banham and Katsaggelos 97]
A model for image distortion

\[ g(x, y) = H[f(x, y)] + \eta(x, y) \]

\[ \rightarrow \text{design restoration filters such that } \hat{f}(x, y) \text{ is as close to } f(x, y) \text{ as possible.} \]
A model for image distortion

- Image restoration
  - Use a priori knowledge of the degradation
  - Modeling the degradation and apply the inverse process
  - Formulate and evaluate objective criteria of goodness
Usual assumptions for the distortion model

- **Noise**
  - Independent of spatial location
    - Exception: periodic noise ...
  - Uncorrelated with image
- **Degradation function H**
  - Linear
  - Position-invariant

\[
g(x, y) = H[f(x, y)] + \eta(x, y)
\]

divide-and-conquer step #1: image degraded only by noise.
Common noise models

- **Gaussian**
  \[ p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\mu)^2/2\sigma^2} \]

- **Rayleigh**
  \[ p(z) = \frac{2}{b} (z-a) e^{-(z-a)^2/b}, \text{ for } z \geq a \]

- **Erlang/Gamma(a,b)**
  \[ p(z) = \frac{a^b z^{b-1}}{(b-1)!} e^{-az}, \text{ for } z \geq 0 \]

- **Exponential**
  \[ p(z) = ae^{-az}, \text{ for } z \geq 0 \]
  $\rightarrow$ additive noise

- **Salt-and-Pepper:** (Impulse)
  \[ p(z) = Pa\delta(z-a) + Pb\delta(z-b) \]

- **Speckle noise:** $a = a_R + ja_I$
  \[ |g(x,y)|^2 \sim |f(x,y)|^2 |a(x,y)|^2 + \eta(x,y) \]
  $a_R, a_I$ zero mean, independent Gaussian
  $\rightarrow$ multiplicative noise on signal magnitude

More details, G&W p313-319
The visual effects of noise

**Figure 5.4** Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

**Figure 5.4 (Continued)** Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.
Recovering from noise

- **overall process**
  Observe and estimate noise type and parameters → apply optimal (spatial) filtering (if known) → observe result, adjust filter type/parameters ...

- **Example noise-reduction filters** [G&W 5.3]
  - Mean/median filter family
  - Adaptive filter family
  - Other filter family
    - e.g. Homomorphic filtering for multiplicative noise [G&W 4.5, Jain 8.13]

\[ f(x, y) \xrightarrow{\text{ln}} \xrightarrow{\text{DFT}} W(u, v) \xrightarrow{(\text{DFT})^{-1}} \xrightarrow{\text{exp}} g(x, y) \]

**FIGURE 4.31** Homomorphic filtering approach
Example: Gaussian noise & mean filter

FIGURE 5.7
(a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascenti, Lixi, Inc.)
Example: salt-and-pepper noise & median filter

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a $7 \times 7$ median filter. (c) Result of adaptive median filtering with $S_{\text{max}} = 7$. 
Recovering from Periodic Noise

FIGURE 5.5
(a) Image corrupted by sinusoidal noise.
(b) Spectrum (each pair of conjugate impulses corresponds to one sine wave).
(Original image courtesy of NASA.)
Recovering from Periodic Noise in Frequency domain

Recall: Butterworth LPF

\[ H(u, v) = \frac{1}{1 + \left[ \frac{D(u, v)}{D_0} \right]^{2n}} \]

Butterworth bandreject filter

\[ H(u, v) = \frac{1}{1 + \left[ \frac{D(u, v)W}{D^2(u, v) - D_0^2} \right]^{2n}} \]

**FIGURE 4.14** (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

**FIGURE 5.15** From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.
Example of bandreject filter

**FIGURE 5.16**
(a) Image corrupted by sinusoidal noise.
(b) Spectrum of (a).
(c) Butterworth bandreject filter (white represents 1).
(d) Result of filtering.
(Original image courtesy of NASA.)
Example of bandpass filter

\[ H_{bp}(u, v) = 1 - H_{br}(u, v) \]

**FIGURE 5.17**
Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.
Notch filter

**FIGURE 5.18**
Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.
Example of notch filter

**FIGURE 4.64**
(a) Sampled newspaper image showing a moiré pattern.
(b) Spectrum.
(c) Butterworth notch reject filter multiplied by the Fourier transform.
(d) Filtered image.
Example of notch filter

FIGURE 5.19
(a) Satellite image of Florida and the Gulf of Mexico showing horizontal scan lines. (b) Spectrum. (c) Notch pass filter superimposed on (b). (d) Spatial noise pattern. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)
Outline

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Recover from linear degradation

- Degradation function
  - Linear (eq 5.5-3, 5.5-4)
    - Homogeneity
    - Additivity
  - Position-invariant (in cartesian coordinates, eq 5.5-5)

→ linear filtering with $H(u,v)$
  convolution with $h(x,y)$ – point spread function

\[ g(x, y) = H[f(x, y)] + \eta(x, y) \]

Divide-and-conquer step #2: linear degradation, noise negligible.
Estimate the degradation Function

- By Image Observation
- By Experimentation
- By modeling
Example of turbulence model

(a) Negligible turbulence.
(b) Severe turbulence, $k = 0.0025$.
(c) Mild turbulence, $k = 0.001$.
(d) Low turbulence, $k = 0.00025$.

(Original image courtesy of NASA.)
Image blur due to motion

(a) Original image.
(b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

G&W Equ. 5.6-11
Image Restoration by Inverse filter

- assume $h$ is known: low-pass filter $H(u,v)$

- inverse filter $\hat{H}(u,v) = 1 / H(u,v)$

- recovered image $\hat{F}(u,v) = G(u,v) \hat{H}(u,v)$
Inverse filtering example
Inverse filtering example

**FIGURE 5.27**
Restoring Fig. 5.25(b) with Eq. (5.7-1).
(a) Result of using the full filter. (b) Result with \( H \) cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

Input image

remedy 1:
inverse filter with cut-off

\[
\hat{H}(u, v) = \begin{cases} 
\frac{1}{H(u, v)}, & |D(u, v)| \leq \varepsilon \\
0, & |D(u, v)| > \varepsilon 
\end{cases}
\]
Inverse filtering under noise

- In reality, we often have
- $H(u, v) = 0$, for some $u$, $v$. e.g. motion blur
- Noise $N(u, v) \neq 0$

\[
\begin{align*}
\hat{H}(u, v) &= \frac{1}{H(u, v)} \\
\hat{F}(u, v) &= \hat{G}(u, v) \hat{H}(u, v) \\
G(u, v) &= F(u, v)H(u, v) + N(u, v) \\
\tilde{F}(u, v) &= F(u, v) + \frac{N(u, v)}{\hat{H}(u, v)}
\end{align*}
\]
Pseudo-inverse filtering

cut-off based on filter frequency

\[ \hat{H}(u, v) = \begin{cases} \frac{1}{H(u, v)}, & |H(u, v)| \geq \varepsilon \\ 0, & |H(u, v)| < \varepsilon \end{cases} \]
Back to the original problem

**FIGURE 5.1 A model of the image degradation/restoration process.**

Inverse filter with cut-off:

\[
\hat{H}(u, v) = \begin{cases} 
1/H(u, v), & |D(u, v)| \leq \varepsilon \\
0, & |D(u, v)| > \varepsilon 
\end{cases}
\]

Pseudo-inverse filter:

\[
\hat{H}(u, v) = \begin{cases} 
1/H(u, v), & |H(u, v)| \geq \varepsilon \\
0, & |H(u, v)| < \varepsilon 
\end{cases}
\]

- Can the filter take values between 1/H(u,v) and zero?
- Can we model noise directly?

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Wiener (mini mean square error) filter

\[ f(x, y) \xrightarrow{\text{Degradation function } H} g(x, y) \xrightarrow{\text{Restoration filter(s)}} \hat{f}(x, y) \]

**DEGRADATION**

**RESTORATION**

\[ W(u, v) \]

- **goal:** restoration with minimum mean-square error (MSE)
  \[ \min_W e^2 = E\{ (f - \hat{f})^2 \} \]
- **optimal solution (nonlinear):**
  \[ \hat{f}(x, y) = E\{ f(x, y) | g(m, n), \forall (m, n) \} \]
- **restrict to linear space-invariant filter**
  \[ \hat{f}(x, y) = w(x, y) * g(x, y) \]
- **find “optimal” linear filter** \( W(u, v) \) with min. MSE ...
Wiener filter

- goal: restoration with minimum mean-square error (MSE)
  \[ \min_W \ E\{ (f - \tilde{f})^2 \} \]
  \[ \tilde{f}(x, y) = w(x, y) \ast g(x, y) \]

- find “optimal” linear filter \( W(u,v) \) with min. MSE

  \( \rightarrow \) orthogonal condition \( E\{ g(f - \tilde{f}) \} = 0 \)
  \( \rightarrow \) correlation function \( R_{fg}(x, y) = w(x, y) \ast R_{gg}(x, y) \)

  \( \rightarrow \) wide-sense-stationary (WSS) signals
  \( R_{fg}(x_1, y_1, x_2, y_2) = E\{ f(x_1, y_1)g(x_2, y_2) \} \xrightarrow{WSS} R_{fg}(x_1 - x_2, y_1 - y_2) \)

  \( \rightarrow \) Fourier Transform: from correlation to spectrum
  \( S_{fg}(u, v) = \mathcal{F}\{ R_{fg}(x, y) \}, \ S_{gg}(u, v) = \mathcal{F}\{ R_{gg}(x, y) \} \)

  \[ W(u, v) = \frac{S_{fg}(u, v)}{S_{gg}(u, v)} = \frac{H^*(u, v)S_{ff}(u, v)}{|H(u, v)|^2S_{ff}(u, v) + S_{\eta\eta}(u, v)} \]

  \( S_{ff} \) and \( S_{\eta\eta} \) are the power spectra of the signal and noise, respectively
SNR and MSE measurement

Signal-to-noise ratio:

\[
SNR = \frac{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |F(u, v)|^2}{\sum_{u=0}^{M-1} \sum_{v=0}^{N-1} |N(u, v)|^2}
\]

Mean square error:

\[
MSE = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} [f(x, y) - \hat{f}(x, y)]^2
\]
Observations about Wiener filter

\[ W(u, v) = \frac{H^*(u, v)S_{ff}(u, v)}{|H(u, v)|^2 S_{ff}(u, v) + S_{\eta\eta}(u, v)} \]

\[ = \frac{1}{H(u, v) + \frac{S_{\eta\eta}}{H^*(u, v)S_{ff}}} \]

- If no noise, \( S_{\eta\eta} \to 0 \)
  \[ W(u, v)|_{S_{\eta\eta} \to 0} = \begin{cases} \frac{1}{H(u, v)}, & H(u, v) \neq 0 \\ 0, & H(u, v) = 0 \end{cases} \]
  \( \rightarrow \) Pseudo inverse filter

- If no blur, \( H(u, v) = 1 \) (Wiener smoothing filter)
  \[ W(u, v)|_{H=1} = \frac{1}{1 + \frac{S_{\eta\eta}(u, v)}{S_{ff}(u, v)}} = \frac{SNR(u, v)}{SNR(u, v) + 1} \]
  \( \rightarrow \) More suppression on noisier frequency bands
1-D Wiener Filter Shape

Wiener Filter implementation

\[ W(u, v) = \frac{H^*(u, v) S_{ff}(u, v)}{|H(u, v)|^2 S_{ff}(u, v) + S_{\eta\eta}(u, v)} \]

\[ = \frac{H^*(u, v)}{|H(u, v)|^2 + \frac{S_{\eta\eta}}{S_{ff}}} \]

\[ = \frac{H^*(u, v)}{|H(u, v)|^2 + K} \]

Where \( K \) is a constant (w.r.t. \( u \) and \( v \)) chosen according to our knowledge of the noise level.
Wiener Filter example

\[ W(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + K} \]
Wiener filter example

**FIGURE 5.28** Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

- Wiener filter is more robust to noise, and preserves high-frequency details.
Wiener filter example

(a) Blurry image (b) restored w. regularized pseudo inverse (c) restored with wiener filter
Wiener filter: when does it not work

How much de-blurring is just enough?

image ‘blurr1’  wiener filter  restored license plate
Improve Wiener filters

- geometric mean filters

\[ W(u, v) = \left[ \frac{H \ast (u, v)}{|H(u, v)^2|} \right]^{\alpha} \left[ \frac{H^*(u, v)}{|H(u, v)|^2 + \beta \frac{S_{yy}(u,v)}{S_{ff}(u,v)}} \right]^{1-\alpha} \]

- Constrained Least Squares
  - Wiener filter emphasizes high-frequency components, while images tend to be smooth

\[
\min_{\hat{f}} |g - H \hat{f}|^2 + \alpha |C \hat{f}|^2
\]

\( \hat{f} \): the estimate for undegraded image

\( C \hat{f} \): a high-passed version of \( \hat{f} \)
**Example of improved wiener filter**

*FIGURE 5.30* Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.
Wavelet Restoration

Two step:
- Fourier-domain inverse filtering
- Wavelet-domain image denoising.

When the blurring function is not invertible, the algorithm is not applicable.
Wavelet-based deconvolution technique for ill-conditioned systems

- Two step:
  - modifying the Wiener filter
    \[ G_\alpha = \frac{H^* S_{xx}}{|H|^2 S_{xx} + \alpha S_{\eta}}. \]
  - Wavelet-domain image denoising.
Example of Wavelet Restoration

Blurred Lena Image
PSNR = 23.2993, MSE = 304.1938

Restored Lena Image (WaRD)
PSNR = 19.5115, MSE = 727.7

Restored Lena Image (Wavelet)
PSNR = 16.8552, MSE = 1341.4

Restored Lena Image (Subband)
PSNR = 20.1223, MSE = 632.2
Power Spectrum Equalization (PSE)

- We assumed that we knew the blurring function $h$, what should we do if they were unknown?
  - Wiener Filter
  
  \[ G = \frac{H^*S_{uu}}{|H|^2S_{uu} + S_{\eta\eta}} \]

- Power Spectrum Equalization -- restore the power spectrum of the degraded image

\[ G = \left[ \frac{S_{uu}}{|H|^2S_{uu} + S_{\eta\eta}} \right]^{1/2} \]
Example of PSE

Lenna after Blurring Plus Noise, Mean Squared Error = 1.0660e+05

Lenna restored using Wiener Filter, Mean Squared Error = 123.2

Lenna restored using PSE, Mean Squared Error = 419.5
Blind deconvolution

- Restore image without knowledge of our blurring function
- Estimate H (Jain approach)
  \[ \log |H| = \frac{1}{M} \sum_{k=1}^{M} [\log |V_k| - \log |\hat{V}_k|] \]
- Wiener filter

http://www.owlnet.rice.edu/~elec539/Projects99/BACH/proj2/blind/bd.html
Example of Blind deconvolution

Lenna after Blurring Plus Noise
Mean Squared Error = 1.0660e+05

Lenna restored using Jain Approach
Mean Squared Error = 283.1
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Geometric distortions

- Modify the spatial relationships between pixels in an image
- a. k. a. “rubber-sheet” transformations

Two basic steps
- Spatial transformation
- Gray-level interpolation

\[ x' = r(x, y) \]
\[ y' = s(x, y) \]
Geometric/spatial distortion examples

(a) Original

(b) Pincushion distortion

(c) Barrel distortion

FIGURE 14.2-1. Example of geometric distortion.
Recovery from geometric distortion

![Recovery from geometric distortion](image)

**FIGURE 5.34** (a) Image showing tiepoints (b) Tiepoints after geometric distortion. (c) Geometrically distorted image, using nearest neighbor interpolation. (d) Restored result. (e) Image distorted using bilinear interpolation. (f) Restored image.
Recovery from geometric distortion

Fig. 5. (c) Image produced by a Computar 2.5mm lens and a Computar 1/3" CCD board camera. (b) Distortion parameters recovered via the minimization of $\xi_3$ are used to map (a) to perspective image. Notice that straight lines in the scene, such as door edges, map to straight lines in the undistorted images.

Omnicamera

http://www.columbia.edu/cu/record/23/20a/omnicamera.html
Estimating distortions

- calibrate
- use flat/edge areas
- ... ongoing work

http://photo.net/learn/dark_noise/

[Tong et. al. ICME2004]
Summary

- A image degradation model
- Restoration from noise
- Restoration from linear degradation
- Inverse and pseudo-inverse filters, Wiener filter, constrained least squares, wavelet restoration,
- Geometric distortions
- Readings
  - G&W Chapter 5.1 – 5.10.
Who said distortion is a bad thing?

blur ...

noise ...

geometric ...

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Suggestions for Final Project -- 1

- Each student or team works on different projects
- Send me by email about the TITLE and a brief description if you will choose the final project by yourself before 10/31/2018 (must be image processing related).
- Two students can work on same project
  - Team work
  - Need to show the contributions for each student
  - Send me by email for the partner you choose before 10/31. Otherwise, I’ll assume you prefer to do the final project by yourself.
Suggestions for Final Project -- 2

For the project:
- Code
- Report
- Presentation
HW2 Presentation